Æfing 3.3. Lausn

1) a) $\int (5x-3)^4 dx$ $\qquad t = 5x-3 \quad \frac{dt}{dx} = 5 \quad dx = \frac{dt}{5}$

$$\Rightarrow \frac{1}{5}\int t^4 dt = \frac{1}{5}\cdot\frac{t^5}{5} + k = \frac{t^5}{25} + k = \underline{\underline{\frac{(5x-3)^5}{25} + k}}$$

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c) $\int x(5x-3)^4 dx$ $\qquad t = 5x-3 \Rightarrow dx = \frac{dt}{5}$

$t = 5x-3$

$\Rightarrow 5x = t+3$

$x = \frac{t+3}{5}$

$$\Rightarrow \int \frac{t+3}{5}\cdot t^4 \cdot \frac{1}{5} dt$$

$$= \frac{1}{25}\int t^5 + 3t^4 dt = \frac{1}{25}\left(\frac{t^6}{6} + \frac{3}{5}t^5\right) + k = \frac{(5x-3)^6}{150} + \frac{(5x-3)^5}{125} + k$$

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e) $\int 15x^2(x^3-2)^4 dx$ $\qquad t = x^3 - 2$

$\frac{dt}{dx} = 3x^2 \Rightarrow dx = \frac{dt}{3x^2}$

$$\Rightarrow \int \frac{15\cancel{x^2}\cdot t^4\, dt}{3\cancel{x^2}} = 5\int t^4 dt = \cancel{5}\frac{t^5}{\cancel{5}} + k = \underline{\underline{(x^3-2)^5 + k}}$$

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g) $\int \cos(5x - \frac{\pi}{8}) dx$ $\qquad t = 5x - \frac{\pi}{8}$

$dt = 5dx \Rightarrow dx = \frac{dt}{5}$

$$\Rightarrow \frac{1}{5}\int \cos(t) dt = \frac{1}{5}\sin(t) + k = \underline{\underline{\frac{1}{5}\sin\left(5x - \frac{\pi}{8}\right) + k}}$$

Æfing 3.3 Lausn

1i)

$$\int 2x^2(x^3+1)^4dx$$

$$t = x^3+1$$

$$dt = 3x^2dx \Rightarrow dx = \frac{dt}{3x^2}$$

$$= \frac{2}{3}\int \frac{\cancel{x^2}}{\cancel{x^2}} \cdot t^4 dt = \frac{2}{3}\cdot\frac{t^5}{5} + K = \underline{\underline{\frac{2(x^3+1)^5}{15} + K}}$$

2a)

$$\int_0^3 \sqrt{x+1}\, dx$$

$$t = x+1$$

$$dt = 1dx$$

$$= \int \sqrt{t}\, dt = \frac{2t^{3/2}}{3} + K$$

$$\Rightarrow \int_0^3 \sqrt{x+1}\, dx = \left[\frac{2(x+1)^{3/2}}{3}\right]_0^3 = \underline{\underline{\frac{14}{3}}} \approx \underline{\underline{4{,}67}}$$

2c)

$$\int_0^1 x\cdot\sqrt{1-x^2}\, dx$$

$$t = 1-x^2$$

$$dt = -2x\,dx \Rightarrow dx = \frac{dt}{-2x}$$

$$\Rightarrow \int_0^1 x\cdot\sqrt{t}\cdot\frac{dt}{-2x} = -\frac{1}{2}\int_0^1 \sqrt{t}\, dt = \left[-\frac{1}{2}\cdot\frac{2t^{3/2}}{3}\right]_0^1 = \left[-\frac{(1-x^2)^{3/2}}{3}\right]_0^1$$

$$= 0 - \left(-\frac{1^{3/2}}{6}\right) = \underline{\underline{\frac{1}{3}}}$$

Øving 3.3 Løsn

2e) $\int_1^e \frac{\ln^5(x)}{x}\,dx$

$t = \ln(x)$

$dt = \frac{1}{x}dx \Rightarrow dx = x\,dt$

$$= \int_1^e \frac{t^5 \cdot \cancel{x}\,dt}{\cancel{x}} = \int_1^e t^5\,dt = \left[\frac{t^6}{6}\right]_1^e = \left[\frac{\ln^6(x)}{6}\right]_1^e$$

$$= \frac{1}{6} - 0 = \underline{\underline{\frac{1}{6}}}$$

3a) $\int 2x\sqrt{x^2+1}\,dx$

$t = x^2+1$

$dt = 2x\,dx$

$dx = \frac{dt}{2x}$

$$\Rightarrow \int \frac{2x}{2x}\sqrt{t}\,dt = \frac{2t^{3/2}}{3} + K = \underline{\underline{\frac{2(x^2+1)^{3/2}}{3} + K}}$$

3c) $\int 2x^3\sqrt{x^4+1}\,dx$

$t = x^4+1$

$dt = 4x^3\,dx$

$dx = \frac{dt}{4x^3}$

$$\Rightarrow \frac{2}{4}\int \frac{x^3}{x^3}\sqrt{t}\,dt = \frac{1}{2}\cdot\frac{2}{3}t^{3/2} + K = \underline{\underline{\frac{1}{3}(x^4+1)^{3/2} + K}}$$

Æfing 3.3 Lausn

3) e)

$$\int (3x^2+2x)e^{(x^3+x^2)}dx$$

$t = x^3+x^2$

$dt = 3x^2+2x\,dx$

$dx = \dfrac{dt}{3x^2+2x}$

$$\Rightarrow \int \frac{(3x^2+2x)}{(3x^2+2x)} e^t dt = \int e^t dt = e^t + K = \underline{\underline{e^{(x^3+x^2)} + K}}$$

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4) a) $\int \sin^4(x)\cos(x)\,dx$

$t = \sin(x)$

$dt = \cos(x)dx$

$dx = \dfrac{dt}{\cos(x)}$

$$\Rightarrow \int t^4 \frac{\cancel{\cos(x)}}{\cancel{\cos(x)}} dt = \int t^4 dt = \frac{t^5}{5} + K = \underline{\underline{\frac{\sin^5(x)}{5} + K}}$$

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c) $\int \sin^5(x)dx = \int \sin^4(x)\cdot\sin(x)dx$

ATH: $\underline{\underline{\sin^2(x) = 1-\cos^2(x)}}$

$$= \int (1-\cos^2(x))^2 \cdot \sin(x)dx$$

$t = \cos(x)$

$dt = -\sin(x)dx$

$dx = \dfrac{dt}{-\sin(x)}$

$$= \int (1-t^2)^2 \cdot \frac{\sin(x)}{-\sin(x)} dt$$

$$= -\int (t^4-2t^2+1)dt = -\left(\frac{t^5}{5} - \frac{2t^3}{3} + t\right) + K = \boxed{-\frac{\cos^5(x)}{5} + \frac{2\cos^3(x)}{3} - \cos(x) + K}$$

4e) $\displaystyle\int \frac{1}{x\sqrt{9+4x^2}}\,dx$ Of langt !!