Æfing 3.2 Lausn

3.21

1) a) $\int x \cdot e^{-x} dx = x \cdot (-e^{-x}) - \int 1 \cdot (-e^{-x}) dx$

$f(x) = x$
$f'(x) = 1$
$g'(x) = e^{-x}$
$g(x) = -e^{-x}$

$= -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + k$

$= \underline{-e^{-x}(x+1) + k}$

1c) $\int x \cdot \sin(x) dx$

$f(x) = x \quad g'(x) = \sin(x)$
$f'(x) = 1 \quad g(x) = -\cos(x)$

$\Rightarrow \int x \sin(x) dx = -x\cos(x) - \int 1 \cdot (-\cos(x)) dx = -x\cos(x) + \int \cos(x) dx$

$= \underline{\sin(x) - x\cos(x) + k}$

1e) $\int x^3 \cos(x) dx$

$f(x) = x^3 \quad g'(x) = \cos(x)$
$f'(x) = 3x^2 \quad g(x) = \sin(x)$

$\Rightarrow \int x^3 \cos(x) dx = x^3 \sin(x) - \underbrace{\int 3x^2 \cdot \sin(x) dx}_{\text{Hlutheildun}}$

$f(x) = x^2 \quad g'(x) = \sin(x)$
$f'(x) = 2x \quad g(x) = -\cos(x)$

$= x^3 \sin(x) - 3\left(x^2 \cdot (-\cos(x)) + 2\underbrace{\int x \cdot \cos(x) dx}_{\text{Hlutheildun}}\right)$

$f(x) = x \quad g'(x) = \cos(x)$
$f'(x) = 1 \quad g(x) = \sin(x)$

$= x^3 \sin(x) + 3x^2 \cos(x) - 6\left(x \cdot \sin(x) - \int \sin(x) dx\right)$

$= \boxed{x^3 \sin(x) + 3x^2 \cos(x) - 6x\sin(x) + 6\cos(x) + k}$

Æfing 3.2

1.g)

Einfalda

$$\int \log(x)\,dx \Rightarrow \int \frac{\ln(x)}{\ln(2)+\ln(5)}\,dx = \frac{1}{\ln(2)+\ln(5)}\int \ln(x)\,dx$$

Hlutheildun

$$\int \ln(x)\,dx = \int \underbrace{\ln(x)}_{f(x)} \cdot \underbrace{1}_{g'(x)}\,dx$$

$f(x) = \ln(x)$ $\quad g'(x) = 1$

$f'(x) = \frac{1}{x}$ $\quad g(x) = x$

$$\Rightarrow \int \ln(x)\cdot 1\,dx = x\ln(x) - \int \frac{1}{x}\cdot x\,dx = x\ln(x) - x + k$$

$$\Leftarrow \text{Svar: } \frac{1}{\ln(2)+\ln(5)}\cdot(x\ln(x) - x + k) = \underline{\underline{\frac{x\ln(x)-x}{\ln(2)+\ln(5)} + k}}$$

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2)a) $\int x\cdot 10^x\,dx$

$f(x) = x$ $\quad g'(x) = 10^x$

$f'(x) = 1$ $\quad g(x) = \frac{10^x}{\ln(10)}$

finna með reiknivél

$$\Rightarrow \int x\cdot 10^x\,dx = \frac{x\cdot 10^x}{\ln(10)} - \frac{1}{\ln(10)}\int 10^x\,dx = \frac{x\cdot 10^x}{\ln(10)} - \frac{1}{\ln(10)}\cdot\frac{10^x}{\ln(10)} + k$$

$$= \frac{x\cdot 10^x}{\ln(10)} - \frac{10^x}{\ln^2(10)} + k$$

$$\underset{=}{\text{Einfalda}} \; \frac{\ln(10)\,x\cdot 10^x - 10^x}{\ln^2(10)} + k$$

Æfing 3.2

ATH: $\boxed{e^{x+7} = e^x \cdot e^7}$

2c) $\int 6xe^{x+7}dx = 6\int x\cdot e^{x+7}dx$

$= 6\int x\cdot e^x\cdot e^7 dx = 6e^7\int x\cdot e^x dx$

Hlutheildun:

$\int xe^x dx$ | $f(x) = x$ $g'(x) = e^x$
$f'(x) = 1$ $g(x) = e^x$

$\int xe^x dx = x\cdot e^x - \int 1\cdot e^x dx = x\cdot e^x - e^x + k$

$\Rightarrow$ SVAR: $6e^7\cdot(xe^x - e^x) + k = \underline{\underline{6(x-1)e^{x+7} + k}}$

2e) $\int \sqrt{x}\ln(x)dx$ | $f(x) = \ln(x)$ $g'(x) = \sqrt{x} = x^{1/2}$
$f'(x) = \frac{1}{x}$ $g(x) = \frac{2x^{3/2}}{3}$

$\Rightarrow \int \sqrt{x}\ln(x)dx = \frac{2x^{3/2}}{3}\cdot\ln(x) - \int \frac{1}{x}\cdot\frac{2x^{3/2}}{3}dx = \frac{2x^{3/2}}{3} - \frac{2}{3}\int x^{1/2}dx$

$= \frac{2x^{3/2}}{3}\cdot\ln(x) - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} + k = \underline{\underline{\frac{2x^{3/2}}{3}\ln(x) - \frac{4x^{3/2}}{9} + k}}$

Æfing 3.2)

2g) $\int \frac{\ln(x+1)}{\sqrt{x+1}}\,dx$

Byrja að $t = x+1$

$\Leftrightarrow \frac{dt}{dx} = 1 \Rightarrow \underline{\underline{dx = dt}}$

$$\Rightarrow \int \frac{\ln(t)}{\sqrt{t}}\,dt = \int \ln(t)\cdot\frac{1}{\sqrt{t}}\,dt$$

$f(t) = \ln(t)$ $g'(t) = \frac{1}{\sqrt{t}}$

$f'(t) = \frac{1}{t}$ $g(t) = 2\cdot\sqrt{t}$

Hlutheildun

$$\Rightarrow \int \frac{\ln(t)}{\sqrt{t}}\,dt = \ln(t)\cdot 2\sqrt{t} - \int \frac{1}{t}\cdot 2t^{1/2}\,dt = \ln(t)\cdot 2\sqrt{t} - 2\int \frac{1}{\sqrt{t}}\,dt$$

$$= 2\ln(t)\sqrt{t} - 4\cdot\sqrt{t} + k = 2\sqrt{t}\left(\ln(t) - 2\right) + k$$

Skipti inn

$$= \underline{\underline{2\sqrt{x+1}\left(\ln(x+1) - 2\right) + k}}$$

Æfing 3.2

4a) $\int_0^{\pi/3} x \cdot \tan^2(x)\,dx$ $\Big|$ $f(x) = x$ $\quad g'(x) = \tan^2(x)$

$f'(x) = 1$ $\quad g(x) = \tan(x) - x$ — Reikni vél

$$\Rightarrow \int_0^{\pi/3} x\tan^2(x)\,dx = \left[x(\tan(x)-x) - \int(\tan(x)-x)\,dx\right]$$

$$= \left[x\tan(x) - x^2 + \ln(\cos(x)) + \frac{x^2}{2}\right]_0^{\pi/3}$$

$$= \left[x\tan(x) + \ln(|\cos(x)|) - \frac{x^2}{2}\right]_0^{\pi/3}$$

$$= \left(\frac{\pi\cdot\sqrt{3}}{3} + \ln(2) - \frac{\left(\frac{\pi}{3}\right)^2}{2}\right) - (0 + 0 - 0)$$

$$= \underline{\underline{\frac{\pi\cdot\sqrt{3}}{3} + \ln\left(\frac{1}{2}\right) - \frac{\pi^2}{18}}}$$

Æfing 3.2 | (6)

4.c) $\displaystyle\int_1^2 (4x^3+3x^2)\ln(x)\,dx$ | $f(x)=\ln(x)$ $g'(x)=4x^3+3x^2$
$f'(x)=\frac{1}{x}$ $g(x)=x^4+x^3$

$$\int_1^2 (4x^3+3x^2)\ln(x)\,dx = \left[\ln(x)\cdot(x^4+x^3)-\int \frac{1}{x}\cdot(x^4+x^3)\,dx\right]_1^2$$

$$= \left[\ln(x)\cdot(x^4+x^3)-\int (x^3+x^2)\,dx\right]_1^2 = \left[\ln(x)\cdot(x^4+x^3)-\frac{x^4}{4}-\frac{x^3}{3}\right]_1^2$$

$$= \left(\ln(2)\cdot 24-4-\frac{8}{3}\right)-\left(\ln(1)\cdot 2-\frac{1}{4}-\frac{1}{3}\right) = \underline{\underline{24\ln(2)-\frac{73}{12}}}$$

$$\approx \underline{\underline{10{,}55}}$$