Æfing 1.2

1) a) $f(x) = 7x^6 - 3x^2 + 2x - 1$

$$\int(7x^6 - 3x^2 + 2x - 1)dx = 7\int x^6 dx - 3\int x^2 dx + 2\int x dx - 1\int dx$$

$$= 7 \cdot \frac{x^{6+1}}{6+1} - 3\frac{x^{2+1}}{2+1} + 2\frac{x^{1+1}}{1+1} - 1 \cdot x + k$$

$$= 7 \cdot \frac{x^7}{7} - 3\frac{x^3}{3} + 2\frac{x^2}{2} - x + k = \underline{\underline{x^7 - x^3 + x^2 - x + k}}$$

---

b) $f(x) = x^6 - x^2 + x - 1$

$$\int(x^6 - x^2 + x - 1)dx = \int x^6 dx - \int x^2 dx + \int x dx - \int dx$$

$$= \underline{\underline{\frac{x^7}{7} - \frac{x^3}{3} + \frac{x^2}{2} - x + k}}$$

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c) $f(x) = e^{-x}$ | $t = -x \Leftrightarrow \frac{dt}{dx} = -1x^{1-1} = -1 \cdot 1 = -1$

$\Leftrightarrow f(t) = e^t$

$$\frac{dt}{dx} = -1 \Leftrightarrow dx = -1dt$$

$$\int e^{-x} dx = \int e^t \cdot (-1) dt = -1 \cdot \int e^t dt = -e^t + k$$

set inn fyrir $t \Leftrightarrow \underline{\underline{\int e^{-x} dx = -e^{-x} + k}}$

Æfing. 1.2

②

d) $f(x) = e^{2-x}$

$\Leftrightarrow \quad f(t) = e^{t}$

$t = 2-x$

$\Leftrightarrow \frac{dt}{dx} = -1$

$\Leftrightarrow dx = -1 \cdot dt$

$\int e^{t} \cdot (-1) dt = -1 \int e^{t} dt = -e^{t} + k$

Skipti: $\int e^{2-x} dx = \underline{\underline{-e^{2-x} + k}}$

---

e) $f(x) = e^{-2x}$

$t = -2x$

$\Leftrightarrow \frac{dt}{dx} = -2$

$\Leftrightarrow dx = -\frac{1}{2} dt$

$f(t) = e^{t}$

$\int e^{t} \cdot (-\frac{1}{2}) dt = -\frac{1}{2} \int e^{t} dt$

$= -\frac{1}{2} e^{t} + k \Leftrightarrow$ Skipti $\int e^{-2x} dx = \underline{\underline{-\frac{1}{2} e^{-2x}}} + k$

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f) $f(x) = 5^{x}$

$\Leftrightarrow f(x) = e^{x \ln(5)}$

$t = x \cdot \ln(5)$

$\Leftrightarrow \frac{dt}{dx} = 1 \cdot \ln(5)$

$\Leftrightarrow dx = \frac{1}{\ln(5)} dt$

$f(t) = e^{t}$

$\frac{1}{\ln(5)} \int e^{t} dt = \frac{e^{t}}{\ln(5)} + k$ Skipti $\rightarrow \int 5^{x} dx = \frac{e^{x \ln(5)}}{\ln(5)} + k$

$= \underline{\underline{\frac{5^{x}}{\ln(5)} + k}}$

③

Æfing 1.2

g) $f(x) = 2^{-x}$

umskrifa

$f(x) = e^{-x\ln(2)}$

$t = -x\ln(2) \Leftrightarrow \frac{dt}{dx} = -1\ln(2)$

$dx = -\frac{1}{\ln(2)}dt$

$f(t) = e^t$

$$-\frac{1}{\ln(2)}\int e^t dt = \frac{-e^t}{\ln(2)} + k \xrightarrow{\text{Skipti}} \frac{-e^{-x\ln(2)}}{\ln(2)} + k \xrightarrow{\text{Einfalda}} \frac{-2^{-x}}{\ln(2)} + k$$

---

h) $f(x) = 3^x \cdot 2^{-x} \xrightarrow{\text{umrita}} f(x) = \frac{3^x}{2^x} \Rightarrow f(x) = \left(\frac{3}{2}\right)^x$

$$\boxed{\text{Regla bls 19} \quad f(x) = a^x \Rightarrow F(x) = \frac{1}{\ln(a)}a^x}$$

$$\Leftrightarrow F(x) = \int \left(\frac{3}{2}\right)^x dx = \frac{1}{\ln\left(\frac{3}{2}\right)} \cdot \left(\frac{3}{2}\right)^x + k$$

Æfing 1.2

2a) $f(x) = \sin(x) \Rightarrow \int \sin(x)dx = \underline{\underline{-\cos(x) + K}}$

---

b) $f(x) = \sin(3x)$

$u = 3x \Rightarrow \frac{du}{dx} = 3$

$\Rightarrow dx = \frac{1}{3}du$

$\frac{1}{3}\int \sin(u)du = -\frac{1}{3}\cos(u) + K$

skipti $\int \sin(3x)dx = -\frac{1}{3}\cos(3x) + K$

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c) $f(x) = \sin(\pi \cdot x)$ Sama og í síðasta dæmi

$\int \sin(\pi x)dx = \underline{\underline{-\frac{\cos(\pi x)}{\pi} + K}}$

---

d) $f(x) = \cos(x) + 2$

$\int (\cos(x) + 2)dx = \int \cos(x)dx + 2\int dx = \underline{\underline{\sin(x) + 2x + K}}$

---

e) $f(x) = \cos(x+2)$

$u = x + 2 \Rightarrow \frac{du}{dx} = 1$

$\int \cos(u)du = \sin(u) + K \xrightarrow{\text{skipti}} \int \cos(x+2)dx = \underline{\underline{\sin(x+2) + K}}$

Æfing 1.2

2 f) $f(x) = 2\cdot\sin(x)\cos(x)$ $\qquad 2\sin(x)\cdot\cos(x) = \sin(2x)$

$\Leftrightarrow f(x) = \sin(2x)$

$$\int \sin(2x)dx = -\frac{\cos(2x)}{2} + k$$

---

g) $f(x) = \cos^2(x)$ $\qquad \cos^2(x) = \frac{1+\cos(2x)}{2} = \frac{1}{2} + \frac{1}{2}\cos(2x)$

$$\Leftrightarrow \int \cos^2(x) = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)dx$$

$$= \frac{x}{2} + \frac{\sin(2x)}{4} + k$$

---

h) $f(x) = \tan(x)$ $\qquad \tan(x) = \frac{\sin(x)}{\cos(x)}$

$u = \cos(x)$

$\frac{du}{dx} = -\sin(x) \Leftrightarrow dx = \frac{1}{-\sin(x)}du$

$$\int \frac{\sin(x)}{\cos(x)}dx = \int \frac{\cancel{\sin(x)}}{u}\cdot\frac{1}{-\cancel{\sin(x)}}du$$

$$= -\int \frac{du}{u} = -\ln|u| + k$$

skipti

$$\int \tan(x)dx = -\ln|\cos(x)| + k$$

Æfing 1.2 |

4) a) $f(x) = x\cdot(x-2) = x^2-2x$

$$\int (x^2-2x)\,dx = \int x^2 dx - 2\int x\,dx = \frac{x^3}{3} - \frac{2x^2}{2} + k = \underline{\underline{\frac{x^3}{3} - x^2 + k}}$$

---

b) $f(x) = x(x-1)(x-2) = x(x^2-2x-x+2) = x^3-2x^2-x^2+2x$

$= x^3-3x^2+2x$

$$\int (x^3-3x^2+2x)\,dx = \int x^3 dx - 3\int x^2 dx + 2\int x\,dx = \frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2} + k$$

$$= \underline{\underline{\frac{x^4}{4} - x^3 + x^2 + k}}$$

---

e) $f(x) = (x-2)^3$

$u = x-2$

$\frac{du}{dx} = 1 \Leftrightarrow dx = du$

$$\int u^3\,du = \frac{u^4}{4} + k \xrightarrow{\text{skipti}} \int (x-2)^3 dx = \frac{(x-2)^4}{4} + k$$

---

f) $f(x) = (5x-2)^3$

$u = 5x-2$

$\frac{du}{dx} = 5 \Leftrightarrow dx = \frac{1}{5}du$

$$\int u^3\,dx = \frac{1}{5}\int u^3\,du = \frac{1}{5}\frac{u^4}{4} + k$$

↙ skipti

$$\int (5x-2)^3 dx = \frac{(5x-2)^4}{20} + k$$