

Finndu 1. afleiðu (diffurkvóta) eftirfarandi falla

a) $f(x) = x^2 - x^3 + 4$

$$f'(x) = 2 \cdot x^{2-1} - 3 \cdot x^{3-1} + 0 = 2x - 3x^2 = \underline{\underline{-2x^2 + 2x}}$$

b) $f(x) = 3x^5 - 5x^3$

$$f'(x) = 5 \cdot 3x^{5-1} - 3 \cdot 5x^{3-1} = \underline{\underline{15x^4 - 15x^2}}$$

c) $f(x) = \frac{1}{2}x^4 - 4\sqrt{x} = \frac{1}{2}x^4 - 4x^{1/2} \leftarrow \text{Umrita}$

$$f'(x) = 4 \cdot \frac{1}{2}x^{4-1} - \frac{1}{2} \cdot 4x^{1/2-1} = 2x^3 - 2x^{-1/2} = 2x^3 - \frac{2}{x^{1/2}} = \underline{\underline{2x^3 - \frac{2}{\sqrt{x}}}}$$

d) $f(x) = 2x - \frac{1}{x} \xrightarrow{\text{Umrita}} f(x) = 2x - x^{-1}$

$$f'(x) = 1 \cdot 2x^{1-1} - (-1) \cdot x^{-1-1} = 2x^0 + x^{-2} = 2 + \frac{1}{x^2}$$

e) $f(x) = \frac{1}{2x^2} - \frac{1}{3x^3} \xrightarrow{\text{Umrita}} f(x) = \frac{1}{2} \cdot x^{-2} - \frac{1}{3} \cdot x^{-3}$

$$f'(x) = -2 \cdot \frac{1}{2} \cdot x^{-2-1} - 3 \cdot \left(-\frac{1}{3}\right) \cdot x^{-3-1} = -x^{-3} + x^{-4} = \underline{\underline{-\frac{1}{x^3} + \frac{1}{x^4}}}$$

f) $f(x) = \frac{4}{\sqrt{x}} + \frac{2}{x^4} \xrightarrow{\text{Umrita}} 4 \cdot x^{-1/2} + 2x^{-4}$

$$f'(x) = -\frac{1}{2} \cdot 4x^{-1/2-1} - 4 \cdot 2x^{-4-1} = \underline{\underline{-\frac{2}{x^{3/2}} - \frac{8}{x^5}}}$$

g) $f(x) = x^3(x^2 - 3) \xrightarrow{\text{Leysi upp síðan}} f(x) = x^3 \cdot x^2 - 3x^3 = x^{3+2} - 3x^3 = x^5 - 3x^3$

$$f'(x) = 5x^{5-1} - 3 \cdot 3x^{3-1} = \underline{\underline{5x^4 - 9x^2}}$$

h) $f(x) = \sqrt{x}(2x^2 + \sqrt{x})$ Leyši upp sviða
 $\implies 2x^2 \cdot x^{1/2} + x^{1/2} \cdot x^{1/2} = 2x^{5/2} + x$

$$f'(x) = \frac{5}{2} \cdot 2 \cdot x^{5/2-1} + 1 \cdot x^{1-1} = 5x^{3/2} + x^0 = \underline{\underline{5x^{3/2} + 1}}$$

j) $f(x) = \frac{x^2+2x}{x-2}$; $g(x) = x^2+2x$; $h(x) = x-2$ Diffurregla

$$f(x) = \frac{g(x)}{h(x)} \implies f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h^2(x)}$$

$$f'(x) = \frac{(2x^{2-1} + 2x^{1-1}) \cdot (x-2) - (x^2+2x) \cdot (x^{1-1} - 0)}{(x-2)^2}$$

$$\Leftrightarrow f'(x) = \frac{(2x+2)(x-2) - (x^2+2x) \cdot 1}{(x-2)^2} = \frac{2x^2-4x+2x-4 - x^2-2x}{(x-2)^2} = \underline{\underline{\frac{x^2-4x-4}{(x-2)^2}}}$$

k) $f(x) = \frac{x^2-1}{x^2+x}$

Sama atferð

$$f(x) = \frac{g(x)}{h(x)} \implies f'(x) = \frac{(2x^{2-1} - 0)(x^2+x) - (x^2-1)(2x^{2-1} + x^{1-1})}{(x^2+x)^2}$$

$$\implies f'(x) = \frac{2x(x^2+x) - (x^2-1)(2x+1)}{(x^2+x)^2} = \frac{(x+1)^2}{(x^2+x)^2}$$

$$\implies f'(x) = \frac{(x+1)^2}{(x^2+x)(x+1) \cdot x} = \frac{x+1}{x^3+x^2} = \frac{(x+1)}{x^2(x+1)} = \underline{\underline{\frac{1}{x^2}}}$$