STÆF3HD05

Kvittun nemanda:	Lausn
Kennitala:	
Kvittun ábyrgðarmanns prófs (tekið utan VMA):	

Skrifaðu nafn þitt og kennitölu í reitinn hér að ofan.

Áfangi:	**Heildun og diffurjöfnur**	**DAGS.:**	**12. des. 2022**
		KL.	**9:00 – 10:30**
Kennari:	**Ingimar Árnason**		
Próflengd:	**90 mínútur**	**Prófsíður:**	**7 (þ.m.t. forsíða)**
Hjálpargögn:	**Reiknivél og skriffæri**	**Fylgiblöð:**	**2**

Leiðbeiningar: Lestu spurningarnar vel áður en þú byrjar að svara, notaðu öll leyfileg hjálpargögn og farðu vandlega yfir úrlausnir þínar áður en þú skilar þeim inn.

SKILAÐU PRÓFBLAÐINU ÁSAMT ÚRLAUSNUM OG VANDAÐU FRÁGANG.

SÝNA SKAL ALLA ÚTREIKNINGA

GANGI ÞÉR VEL!

$\sin(2x) = 2\sin(x)\cos(x)$

1. (25%) Reiknaðu þessi heildi:

a) $\int \sin(x)\cos(x)dx$ $= \frac{1}{2}\int \sin(2x)dx = \frac{1}{2}\left(-\frac{\cos(2x)}{2}\right) + k = -\frac{1}{4}\cos(2x) + k$

eða innsetning:

$u = \sin(x)$

$du = \cos(x)dx$

$\int u\,du = \frac{u^2}{2} + k = \frac{\sin^2(x)}{2} + k$

Diffrun á svörum:

$\frac{\sin^2(x)}{2} \to \sin(x)\cdot\cos(x)$

$-\frac{1}{4}\cos(2x) \to -\frac{1}{4}(-\sin(2x))\cdot 2 = \frac{\sin(2x)}{2} = \sin(x)\cdot\cos(x)$

b) $\int \frac{x+3}{x^2+6x+8}dx$ $= \int\left(\frac{\frac{1}{2}}{x+2} + \frac{\frac{1}{2}}{x+4}\right)dx = \frac{1}{2}\ln(x+2) + \frac{1}{2}\ln(x+4) + k$

$\frac{x+3}{(x+2)(x+4)} = \frac{A}{x+2} + \frac{B}{x+4} = \frac{A(x+4) + B(x+2)}{(x+2)(x+4)}$

x: $Ax + Bx \to A + B = 1 \quad B = 1 - A$

3: $4A + 2B \to 4A + 2B = 3 \quad 4A + 2(1-A) = 3$

$4A + 2 - 2A = 3$

$2A = 1 \quad A = \frac{1}{2} \quad B = \frac{1}{2}$

c) $\int \frac{(\sqrt{x}+3)^3}{\sqrt{x}}dx$ $= 2\int u^3 du = 2\cdot\frac{u^4}{4} + k = \frac{(\sqrt{x}+3)^4}{2} + k$

$u = \sqrt{x} + 3$

$du = \frac{1}{2\sqrt{x}}dx$

d) $\int x \cdot e^{2x} \mathrm{dx}$ $= x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + k$

$f = x \quad g = e^{2x}$

$f' = dx \quad G = \frac{e^{2x}}{2}$

e) $\int_1^{\sqrt{3}} \frac{3}{1+x^2} \mathrm{dx}$ $= 3\arctan(x)\Big]_1^{\sqrt{3}} = 3(\arctan(\sqrt{3}) - \arctan(1))$

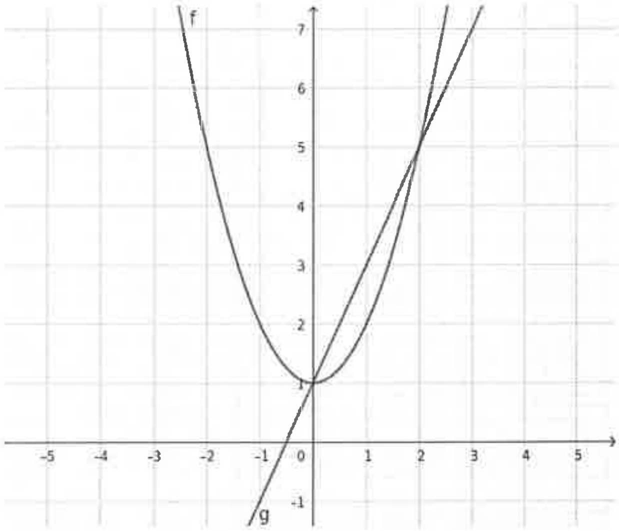
$= 3\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \pi - \frac{3}{4}\pi$

$= \frac{\pi}{4}$

2. (25%) Svæði afmarkast af ferlum fallanna: $f(x) = x^2 + 1$ og $g(x) = 2x + 1$

a) (5%) Reiknaðu út skurðpunkta ferlana.



$x^2 + 1 = 2x + 1$

$x^2 - 2x = 0$

$x = 0 \quad y = 1$

$x = 2 \quad y = 5$

skp eru: $(0,1) \wedge (2,5)$

b) (10%) Reiknaðu flatarmál svæðisins.

$A = \int ((2x+1) - (x^2+1))\,dx = \int (-x^2 + 2x)\,dx$

$\left[-\frac{x^3}{3} + x^2 \right]_0^2 = -\frac{8}{3} + 4 - 0 = \frac{12-8}{3} = 4/3$

c) (10)% Reiknaðu rúmmál snúðsins sem myndast ef svæðinu er snúið hring um x-ás.

$V = \pi \int_0^2 ((2x+1)^2 - (x^2+1)^2)\,dx$

$4x^2 + 4x + 1 - (x^4 + 2x^2 + 1) = -x^4 + 2x^2 + 4x$

$V = \pi \int (-x^4 + 2x^2 + 4x)\,dx = \pi \left[-\frac{x^5}{5} + \frac{2x^3}{3} + 2x^2 \right]_0^2 =$

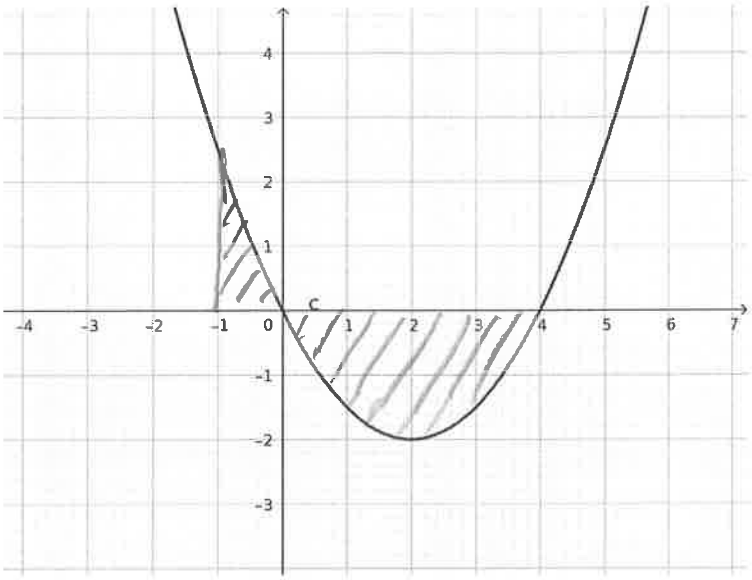
$\pi \left(-\frac{2^5}{5} + \frac{2}{3} \cdot 2^3 + 2 \cdot 2^2 - 0 \right) = \frac{104}{15}\pi$

3. (15%) Grafið sýnir feril $f(x) = \frac{1}{2}x^2 - 2x$

Reiknaðu út flatarmálið sem afmarkast af ferlinum

og x-ás á bilinu $-1 \leq x \leq 4$.

Sýndu svæðið á myndinni.



$$A_1 = \int_{-1}^{0} \left(\tfrac{1}{2}x^2 - 2x\right)dx = \int_{0}^{-1} \left(2x - \tfrac{1}{2}x^2\right)dx = \left[x^2 - \frac{x^3}{6}\right]_0^{-1} =$$

$$(-1)^2 - \frac{(-1)^3}{6} = 1 + \frac{1}{6} = \frac{7}{6}$$

$$A_2 = \int_{0}^{4} \left(\tfrac{1}{2}x^2 - 2x\right)dx = \left[\frac{x^3}{6} - x^2\right]_0^4 = \frac{4^3}{6} - 4^2 - 0$$

$$= \frac{32}{3} - 16 = -\frac{16}{3}$$

$$A = |A_1| + |A_2| = \frac{7}{6} + \frac{16}{3} = \frac{39}{6} = \frac{13}{2}$$

4. (10%)
Um mismunarununa $\{a_i\}$ gildir að $a_1 = 4$ og $a_{13} = 34$.
Ritaðu fjóra fyrstu liði rununnar og finndu síðan s_{17}.

$a_{13} = a_1 + (13-1) \cdot d$

$34 = 4 + 12 \cdot d$

$30 = 12d$

$d = \frac{30}{12} = 2,5$

4, 6,5, 9, 11,5, 13

$S_{17} = n \frac{2a_1 + (n-1)d}{2} = 17 \cdot \frac{2 \cdot 4 + (17-1) \cdot 2,5}{2}$

$= 408$

5. (10%) Leystu þessa diffurjöfnu fyrir y.

$$\frac{dy}{dx} = xy^3$$

$\int \frac{dy}{y^3} = \int x\,dx$

$\int y^{-3} dy = \int x\,dx$

$\frac{y^{-2}}{-2} = \frac{x^2}{2} + k$

$y^{-2} = -x^2 + k$

$\frac{1}{y^2} = -x^2 + k$

$y^2 = -\frac{1}{x^2} + k$

$y = \sqrt{-\frac{1}{x^2}} + k$

6. (5%) Diffraðu:

$f(x) = x \cdot \arctan(\sqrt{x})$

$$f'(x) = 1 \cdot \arctan(\sqrt{x}) + x \cdot \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$$

$$= \arctan(\sqrt{x}) + \frac{x}{1+x} \cdot \frac{1}{2\sqrt{x}}$$

7. (5%) Finndu með óbeinni diffrun dy/dx:

$x^3 - 2\text{xy} + y^2 = 6x$

$$3x^2dx - 2dxy - 2xdy + 2ydy = 6dx$$

$$-2xdy + 2ydy = 6dx + 3x^2dx - 2ydx$$

$$dy(2y - 2x) = (6 + 3x^2 - 2y)dx$$

$$\frac{dy}{dx} = \frac{6 + 3x^2 - 2y}{2y - 2x}$$

8. (5%) Sýndu fram á að:

$\int \ln(x)\text{dx} = x(\ln(x) - 1) + k$

$$\int \ln(x)\,dx = x \cdot \ln(x) - \int \frac{1}{x} x\,dx = x \cdot \ln(x) - \int dx$$

$f = \ln(x)$ $g = dx$

$f' = \frac{1}{x}$ $G = x$

$$= x \cdot \ln(x) - x + k$$

$$x(\ln(x) - 1) + k$$

eða diffra:

$$x\ln(x) - x \to 1\,\ln(x) + x\frac{1}{x} - 1 = \ln(x) + 1 - 1$$

$$= \ln(x)$$

Afleiður nokkurra falla

$f(x)=x^n$	$f'(x)=nx^{n-1}$
$f(x)=\sqrt{x}$	$f'(x)=\dfrac{1}{2\sqrt{x}}$
$f(x)=\sin(x)$	$f'(x)=\cos(x)$
$f(x)=\cos(x)$	$f'(x)=-\sin(x)$
$f(x)=\tan(x)$	$f'(x)=1+\tan^2(x)$
$f(x)=\tan(x)$	$f'(x)=\dfrac{1}{\cos^2(x)}$
$f(x)=\cot(x)$	$f'(x)=-(1+\cot^2(x))$
$f(x)=\cot(x)$	$f'(x)=\dfrac{-1}{\sin^2(x)}$
$f(x)=\ln(x)$	$f'(x)=\dfrac{1}{x}$
$f(x)=\log_a(x)$	$f'(x)=\dfrac{1}{x\cdot\ln(a)}$
$f(x)=e^x$	$f'(x)=e^x$
$f(x)=a^x$	$f'(x)=a^x\ln(a)$
$f(x)=\arcsin(x)$	$f'(x)=\dfrac{1}{\sqrt{1-x^2}}$
$f(x)=\arctan(x)$	$f'(x)=\dfrac{1}{1+x^2}$

Diffrunarreglur

$y=k\cdot f(x)$	$y'=k\cdot f'(x)$
$y=f(x)+g(x)$	$y'=f'(x)+g'(x)$
$y=f(x)\cdot g(x)$	$y'=f'(x)\cdot g(x)+f(x)\cdot g'(x)$
$y=\dfrac{f(x)}{g(x)}$	$y'=\dfrac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$
$y=f(g(x))$	$y'=f'(g(x))\cdot g'(x)$

Reglur um diffur

$d(f\cdot g)=df\cdot g+f\cdot dg$	$d\left(\dfrac{f}{g}\right)=\dfrac{df\cdot g-f\cdot dg}{g^2}$
$d(f\pm g)=df\pm dg$	$d(f(g(x)))=f'(g(x))\cdot dg(x)$

Línuleg nálgun

$$l(x)=f(x_0)+f'(x_0)(x-x_0)$$

Stofnföll nokkurra falla

$\int x^n dx=\dfrac{x^{n+1}}{n+1}+K$	$\int \sin(x)dx=-\cos(x)+K$	$\int \cos(x)dx=\sin(x)+K$
$\int (1+\tan^2(x))dx=\tan(x)+K$	$\int (1+\cot^2(x))dx=-\cot(x)+K$	$\int \sin^2(x)dx=\dfrac{x}{2}-\dfrac{\sin(2x)}{4}+K$
$\int \tan^2(x)dx=\tan(x)-x+K$	$\int \cot^2(x)dx=-\cot(x)-x+K$	$\int \cos^2(x)dx=\dfrac{x}{2}+\dfrac{\sin(2x)}{4}+K$
$\int e^x dx=e^x+K$	$\int a^x dx=\dfrac{a^x}{\ln(a)}+K$	$\int \dfrac{1}{x}dx=\ln\lvert x\rvert+K$
$\int \dfrac{1}{1+x^2}dx=\arctan(x)+K$	$\int \dfrac{1}{\sqrt{1-x^2}}dx=\arcsin(x)+K$	

Heildunarreglur

$\int f(x)dx=F(x)+K,\qquad F'(x)=f(x)$	$\int_a^b f(x)dx=\left[F(x)\right]_a^b=F(b)-F(a)$
$\int k\cdot f(x)dx=k\int f(x)dx$	$\int_a^b k\cdot f(x)dx=k\int_a^b f(x)dx$
$\int (-f(x))dx=-\int f(x)dx$	$\int_a^a f(x)dx=0$
$\int (f(x)+g(x))dx=\int f(x)dx+\int g(x)dx$	$\int_a^b f(x)dx=-\int_b^a f(x)dx$

$\int_a^b f(x)dx = f(c)(b-a)$	$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

Flatarmál – Rúmmál

$A = \int_a^b f(x)dx = \Big[F(x)\Big]_a^{+}$	$A = \int_a^b (f(x)-g(x))dx = \int_a^b (EF - NF)dx$
$V = \int_a^b \pi (f(x))^2 dx = \int_a^b \pi \cdot R^2 dx$	$V = \int_a^b \pi ((f(x))^2 - (g(x))^2)dx = \int_a^b \pi \cdot (R^2 - r^2)dx$

Heildunaraðferðir

$$\int f(g(x))\cdot g'(x)dx = \int f(t)dt = F(t) + K = F(g(x)) + K$$

$$\int f\cdot dg = fg - \int g\cdot df$$

$t = g(x)$ og $dt = g'(x)dx$

$$\int_a^b f(g(x))\cdot g'(x)dx = \int_{g(a)}^{g(b)} f(t)dt = \Big[F(t)\Big]_{g(a)}^{g(b)}$$

$t = g(x) \qquad x = b,\ t = g(b)$

$dt = g'(x)dx \qquad x = a,\ t = g(a)$

$$Ax^2 + Bx + C = 0$$
$$D = B^2 - 4AC$$
$$x = \frac{-B \pm \sqrt{D}}{2A}$$

Diffurjöfnur

$$\frac{dy}{dx} = f(x) \Leftrightarrow y = F(x) + K$$

$$\frac{dy}{dx} = f(x)\cdot g(y) \Leftrightarrow \int \frac{1}{g(y)}dy = \int f(x)dx$$

Reiknireglur logra

$\ln(ab) = \ln(a) + \ln(b)$	$\ln(\frac{a}{b}) = \ln(a) - \ln(b)$	$\ln(a^n) = n\ln(a)$	$\ln(1) = 0$ $\ln(e) = 1$	$\log_a(x) = \frac{\ln(x)}{\ln(a)}$

Andhverf hornaföll

$\sin(\arcsin(x)) = x$	$\cos(\arccos(x)) = x$	$\tan(\arctan(x)) = x$	$\cot(\text{arccot}(x)) = x$
$\arcsin(\sin(x)) = x$	$\arccos(\cos(x)) = x$	$\arctan(\tan(x)) = x$	$\text{arccot}(\cot(x)) = x$
$\arcsin(-x) = -\arcsin(x)$	$\arccos(-x) = \pi - \arccos(x)$	$\arctan(-x) = -\arctan(x)$	$\text{arccot}(-x) = -\text{arccot}(x)$
$\arcsin(x) + \arccos(x) = \frac{\pi}{2}$	$\arctan(x) + \text{arccot}(x) = \frac{\pi}{2}$		

	$y = \arcsin(x)$	$y = \arccos(x)$	$y = \arctan(x)$	$y = \text{arccot}(x)$
Skilgr.mengi	$-1 \le x \le 1$	$-1 \le x \le 1$	$\infty < x < \infty$	$\infty < x < \infty$
Myndmengi	$-\pi/2 \le y \le \pi/2$	$0 \le y \le \pi$	$-\pi/2 < y < \pi/2$	$0 < y < \pi$

Summur - Runur - Raðir

$\sum_{i=k}^{n} a_i = a_k + a_{k+1} + \ldots + a_n$		
$s_I = \sum_{i=1}^{n} k_i \cdot \Delta x_i$	$S_I = \sum_{i=1}^{n} K_i \cdot \Delta x_i$	$\sigma_I = \sum_{i=1}^{n} f(t_i)\cdot \Delta x_i$
Runa $a_1, a_2, a_3, \ldots, a_i, \ldots$	Röð $\sum_{i=1}^{\infty} a_i = a_1 + a_2 + a_3 + \ldots$	

Mismunaruna

$a_{i+1} = a_i + d$	$a_i = a_1 + (i-1)d$	$s_n = n\cdot\frac{a_1 + a_n}{2}$	$s_n = n\cdot\frac{2a_1 + (n-1)d}{2}$

Kvótaruna

$a_{i+1} = a_i \cdot q$	$a_i = a_1 \cdot q^{i-1}$	$s_n = a_1\cdot\frac{1-q^n}{1-q}$	$s_n = a_1\cdot\frac{q^n - 1}{q-1}$

Hornafræði

Skilgreiningar

Sinus: sin(v) Cosinus: cos(v) Tangens: tan(v) Cotangens: cot(v)

$$\cos(v)=\frac{\text{aðlæg hlið}}{\text{langhlið}} \qquad \sin(v)=\frac{\text{mótlæg hlið}}{\text{langhlið}} \qquad \tan(v)=\frac{\text{mótlæg hlið}}{\text{aðlæg hlið}}=\frac{\sin(v)}{\cos(v)}=\frac{1}{\cot(v)}$$

Umskriftarreglur

$\sin(-v)=-\sin(v)$	$\cos(-v)=\cos(v)$	$\tan(-v)=-\tan(v)$	$\cot(-v)=-\cot(v)$
$\sin(v+\pi)=-\sin(v)$	$\cos(v+\pi)=-\cos(v)$	$\tan(v+\pi)=\tan(v)$	$\cot(v+\pi)=\cot(v)$
$\sin(\pi-v)=\sin(v)$	$\cos(\pi-v)=-\cos(v)$	$\tan(\pi-v)=-\tan(v)$	$\cot(\pi-v)=-\cot(v)$
$\sin(v+\pi/2)=\cos(v)$	$\cos(v+\pi/2)=-\sin(v)$	$\tan(v+\pi/2)=-\cot(v)$	$\cot(v+\pi/2)=-\tan(v)$
$\sin(v-\pi/2)=-\cos(v)$	$\cos(v-\pi/2)=\sin(v)$	$\tan(v-\pi/2)=-\cot(v)$	$\cot(v-\pi/2)=-\tan(v)$
$\sin(\pi/2-v)=\cos(v)$	$\cos(\pi/2-v)=\sin(v)$	$\tan(\pi/2-v)=\cot(v)$	$\cot(\pi/2-v)=\tan(v)$

$$\cos(v)=\frac{\pm 1}{\sqrt{1+\tan^2(v)}} \qquad \sin(v)=\frac{\pm\tan(v)}{\sqrt{1+\tan^2(v)}}$$

$$\sin^2(v)+\cos^2(v)=1 \qquad \frac{1}{\cos^2(v)}=1+\tan^2(v) \qquad \frac{1}{\sin^2(v)}=1+\cot^2(v)$$

Tvöfalt horn

$$\sin(2v)=2\sin(v)\cdot\cos(v)$$

$$\cos(2v)=\cos^2(v)-\sin^2(v) \qquad \cos(2v)=2\cos^2(v)-1 \qquad \cos(2v)=1-2\sin^2(v)$$

$$\tan(2v)=\frac{2\tan(v)}{1-\tan^2(v)} \qquad \cos^2(v)=\frac{1+\cos(2v)}{2} \qquad \sin^2(v)=\frac{1-\cos(2v)}{2}$$

Summuformúlur

$$\sin(u+v)=\sin(u)\cos(v)+\cos(u)\sin(v), \qquad \sin(u-v)=\sin(u)\cos(v)-\cos(u)\sin(v)$$

$$\cos(u+v)=\cos(u)\cos(v)-\sin(u)\sin(v), \qquad \cos(u-v)=\cos(u)\cos(v)+\sin(u)\sin(v),$$

$$\tan(u+v)=\frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}, \qquad \tan(u-v)=\frac{\tan(u)-\tan(v)}{1+\tan(u)\tan(v)}$$

Liðunarformúlur

$$\sin(u)\cos(v)=\tfrac{1}{2}(\sin(u+v)+\sin(u-v)) \qquad \sin(u)\sin(v)=-\tfrac{1}{2}(\cos(u+v)-\cos(u-v))$$

$$\cos(u)\cos(v)=\tfrac{1}{2}(\cos(u+v)+\cos(u-v))$$

Þáttunarformúlur

$$\sin(s)+\sin(t)=2\sin\left(\frac{s+t}{2}\right)\cdot\cos\left(\frac{s-t}{2}\right) \qquad \cos(s)+\cos(t)=2\cos\left(\frac{s+t}{2}\right)\cdot\cos\left(\frac{s-t}{2}\right)$$

$$\sin(s)-\sin(t)=2\cos\left(\frac{s+t}{2}\right)\cdot\sin\left(\frac{s-t}{2}\right) \qquad \cos(s)-\cos(t)=-2\sin\left(\frac{s+t}{2}\right)\cdot\sin\left(\frac{s-t}{2}\right)$$

Kósínusreglan

$$a^2=b^2+c^2-2bc\cdot\cos(A)$$

Sínusreglan

$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}=2R$$

Lausn á hornafallajöfnu

$\sin(v)=k$

$$v=\begin{cases}\sin^{-1}(k)+360\cdot h\\ 180-\sin^{-1}(k)+360\cdot h\end{cases}$$

$\cos(v)=k$

$$v=\begin{cases}\cos^{-1}(k)+360\cdot h\\ -\cos^{-1}(k)+360\cdot h\end{cases}$$

$\tan(v)=k$

$$v=\tan^{-1}(k)+180\cdot h$$

Gildi hornafalla

Horn v		sin(v)	cos(v)	tan(v)	cot(v)	sec(v)	csc(v)
Gráður	Rad.						
0°	0	0	1	0	X	1	X
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$2/\sqrt{3}$	2
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\sqrt{3}/3$	2	$2/\sqrt{3}$
90°	$\pi/2$	1	0	X	0	X	1
120°	$2\pi/3$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$	$-\sqrt{3}/3$	-2	$2/\sqrt{3}$
135°	$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$5\pi/6$	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}/3$	$-\sqrt{3}$	$-2/\sqrt{3}$	2
180°	π	0	-1	0	X	-1	X
210°	$7\pi/6$	$-1/2$	$-\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$	$-2/\sqrt{3}$	-2
225°	$5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240°	$4\pi/3$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$	$\sqrt{3}/3$	-2	$-2/\sqrt{3}$
270°	$3\pi/2$	-1	0	X	0	X	-1
300°	$5\pi/3$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$	$-\sqrt{3}/3$	2	$-2/\sqrt{3}$
315°	$7\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
330°	$11\pi/6$	$-1/2$	$\sqrt{3}/2$	$-\sqrt{3}/3$	$-\sqrt{3}$	$2/\sqrt{3}$	-2
360°(=0°)	2π	0	1	0	X	1	X

X: Ekki til gildi af hornafallinu

$$\sec(v) = \frac{1}{\cos(v)}, \qquad \csc(v) = \frac{1}{\sin(v)}$$

$\sqrt{2} = 1{,}4142$ $\sqrt{3} = 1{,}7321$ $\sqrt{2}/2 = 0{,}7071$ $\sqrt{3}/2 = 0{,}8660$ $\sqrt{3}/3 = 0{,}5774$