

# Hornafraeði

## Skilgreiningar

Sinus:  $\sin(v)$

Cosinus:  $\cos(v)$

Tangens:  $\tan(v)$

Cotangens:  $\cot(v)$

$$\cos(v) = \frac{\text{aðlæg hlið}}{\text{langhlið}}$$

$$\sin(v) = \frac{\text{mótlæg hlið}}{\text{langhlið}}$$

$$\tan(v) = \frac{\text{mótlæg hlið}}{\text{aðlæg hlið}} = \frac{\sin(v)}{\cos(v)} = \frac{1}{\cot(v)}$$

## Umskriftarreglur

$$\sin(-v) = -\sin(v)$$

$$\cos(-v) = \cos(v)$$

$$\tan(-v) = -\tan(v)$$

$$\cot(-v) = -\cot(v)$$

$$\sin(v+\pi) = -\sin(v)$$

$$\cos(v+\pi) = -\cos(v)$$

$$\tan(v+\pi) = \tan(v)$$

$$\cot(v+\pi) = \cot(v)$$

$$\sin(\pi-v) = \sin(v)$$

$$\cos(\pi-v) = -\cos(v)$$

$$\tan(\pi-v) = -\tan(v)$$

$$\cot(\pi-v) = -\cot(v)$$

$$\sin(v+\pi/2) = \cos(v)$$

$$\cos(v+\pi/2) = -\sin(v)$$

$$\tan(v+\pi/2) = -\cot(v)$$

$$\cot(v+\pi/2) = -\tan(v)$$

$$\sin(v-\pi/2) = -\cos(v)$$

$$\cos(v-\pi/2) = \sin(v)$$

$$\tan(v-\pi/2) = -\cot(v)$$

$$\cot(v-\pi/2) = -\tan(v)$$

$$\sin(\pi/2-v) = \cos(v)$$

$$\cos(\pi/2-v) = \sin(v)$$

$$\tan(\pi/2-v) = \cot(v)$$

$$\cot(\pi/2-v) = \tan(v)$$

$$\cos(v) = \frac{\pm 1}{\sqrt{1+\tan^2(v)}}$$

$$\sin(v) = \frac{\pm \tan(v)}{\sqrt{1+\tan^2(v)}}$$

$$\sin^2(v) + \cos^2(v) = 1$$

$$\frac{1}{\cos^2(v)} = 1 + \tan^2(v)$$

$$\frac{1}{\sin^2(v)} = 1 + \cot^2(v)$$

## Tvöfalt horn

$$\sin(2v) = 2\sin(v) \cdot \cos(v)$$

$$\cos(2v) = \cos^2(v) - \sin^2(v)$$

$$\cos(2v) = 2\cos^2(v) - 1$$

$$\cos(2v) = 1 - 2\sin^2(v)$$

$$\tan(2v) = \frac{2\tan(v)}{1 - \tan^2(v)}$$

$$\cos^2(v) = \frac{1 + \cos(2v)}{2}$$

$$\sin^2(v) = \frac{1 - \cos(2v)}{2}$$

## Summuformúlur

$$\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v),$$

$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v),$$

$$\tan(u+v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)},$$

$$\sin(u-v) = \sin(u)\cos(v) - \cos(u)\sin(v),$$

$$\cos(u-v) = \cos(u)\cos(v) + \sin(u)\sin(v),$$

$$\tan(u-v) = \frac{\tan(u) - \tan(v)}{1 + \tan(u)\tan(v)}$$

## Liðunarformúlur

$$\sin(u)\cos(v) = \frac{1}{2}(\sin(u+v) + \sin(u-v))$$

$$\cos(u)\cos(v) = \frac{1}{2}(\cos(u+v) + \cos(u-v))$$

$$\sin(u)\sin(v) = -\frac{1}{2}(\cos(u+v) - \cos(u-v))$$

## Þáttunarformúlur

$$\sin(s) + \sin(t) = 2\sin\left(\frac{s+t}{2}\right) \cdot \cos\left(\frac{s-t}{2}\right)$$

$$\sin(s) - \sin(t) = 2\cos\left(\frac{s+t}{2}\right) \cdot \sin\left(\frac{s-t}{2}\right)$$

$$\cos(s) + \cos(t) = 2\cos\left(\frac{s+t}{2}\right) \cdot \cos\left(\frac{s-t}{2}\right)$$

$$\cos(s) - \cos(t) = -2\sin\left(\frac{s+t}{2}\right) \cdot \sin\left(\frac{s-t}{2}\right)$$

## Kósínusreglan

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$

## Sínusreglan

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} = 2R$$

## Lausn á hornafallajöfnu

$$\sin(v) = k$$

$$v = \begin{cases} \sin^{-1}(k) + 360 \cdot h \\ 180 - \sin^{-1}(k) + 360 \cdot h \end{cases}$$

$$\cos(v) = k$$

$$v = \begin{cases} \cos^{-1}(k) + 360 \cdot h \\ -\cos^{-1}(k) + 360 \cdot h \end{cases}$$

$$\tan(v) = k$$

$$v = \tan^{-1}(k) + 180 \cdot h$$

# Gildi hornafalla

Horn $v$		$\sin(v)$	$\cos(v)$	$\tan(v)$	$\cot(v)$	$\sec(v)$	$\csc(v)$
Gráður	Rad.						
$0^\circ$	0	0	1	0	X	1	X
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$45^\circ$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60^\circ$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2}{\sqrt{3}}$
$90^\circ$	$\frac{\pi}{2}$	1	0	X	0	X	1
$120^\circ$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2}{\sqrt{3}}$
$135^\circ$	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$150^\circ$	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	2
$180^\circ$	$\pi$	0	-1	0	X	-1	X
$210^\circ$	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	-2
$225^\circ$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
$240^\circ$	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2}{\sqrt{3}}$
$270^\circ$	$\frac{3\pi}{2}$	-1	0	X	0	X	-1
$300^\circ$	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2}{\sqrt{3}}$
$315^\circ$	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
$330^\circ$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2}{\sqrt{3}}$	-2
$360^\circ(=0^\circ)$	$2\pi$	0	1	0	X	1	X

X: Ekki til gildi af hornafallinu

$$\sec(v) = \frac{1}{\cos(v)}, \quad \csc(v) = \frac{1}{\sin(v)}$$

$$\sqrt{2} = 1,4142$$

$$\sqrt{3} = 1,7321$$

$$\frac{\sqrt{2}}{2} = 0,7071$$

$$\frac{\sqrt{3}}{2} = 0,8660$$

$$\frac{\sqrt{3}}{3} = 0,5774$$