

Nafn: Lausn

Einkunn: _____

VMA haust 2018 STÆF2RH05 kaflapróf 2 úr köflum 4 og 5

1. (20%) Reiknaðu út stærð boganna AB, CE og stærð hornanna x og y. Miðpunktur hringins er O. Boginn AC er 100° , boginn EF er 30° og boginn DF er 102° .

Hornið x = 101°

Hornið y = 39°

Boginn \widehat{AB} = 50°

Boginn \widehat{CE} = 30°

- ① Finnum \widehat{AB} með topphornareglunni:

$A = \frac{y+x}{2}$ (bls. 66) y og x eru bogalengdir

$\frac{\widehat{AB} + 30^\circ}{2} = 40^\circ \cdot 2$

$\widehat{AB} + 30^\circ = 80^\circ$
 $-30^\circ \quad -30^\circ$

$\widehat{AB} = 50^\circ$

- ② Finnum \widehat{CE} :

$\widehat{AB} + \widehat{AC} + \widehat{CE} = 180^\circ$

$50^\circ + 100^\circ + \widehat{CE} = 180^\circ$

$150^\circ + \widehat{CE} = 180^\circ \rightarrow \widehat{CE} = 180^\circ - 150^\circ = \underline{30^\circ}$ * hornasumma hálfhrings

- ③ Finnum x með topphornareglunni:

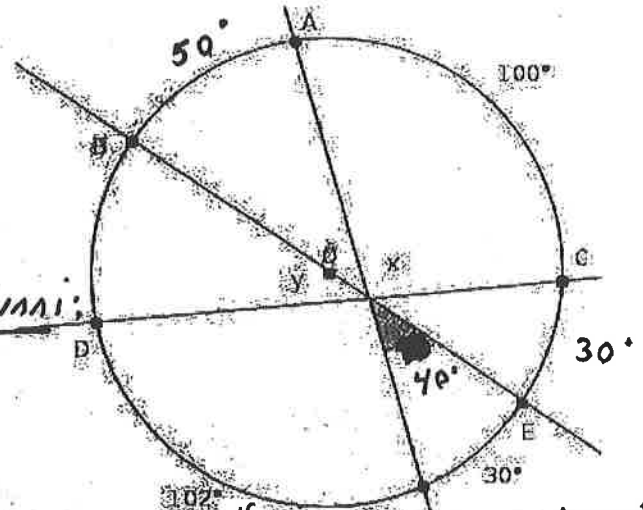
$x = \frac{100^\circ + 102^\circ}{2} = \frac{202^\circ}{2} = \underline{101^\circ}$

- ④ Finnum y:

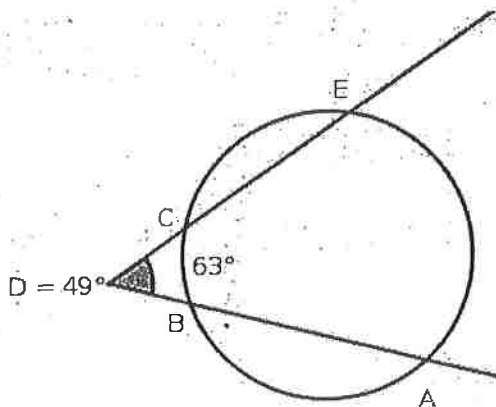
$y + x + 40^\circ = 180^\circ$ gránhorn á beinni línu

$y + 101^\circ + 40^\circ = 180^\circ$

$y + 141^\circ = 180^\circ \rightarrow y = 180^\circ - 141^\circ = \underline{39^\circ}$



2. (5%) Reiknaðu út stærð bogans AE.



Boginn AE = 161°

Reglan um horn utan við hring

$A = \frac{y-x}{2}$

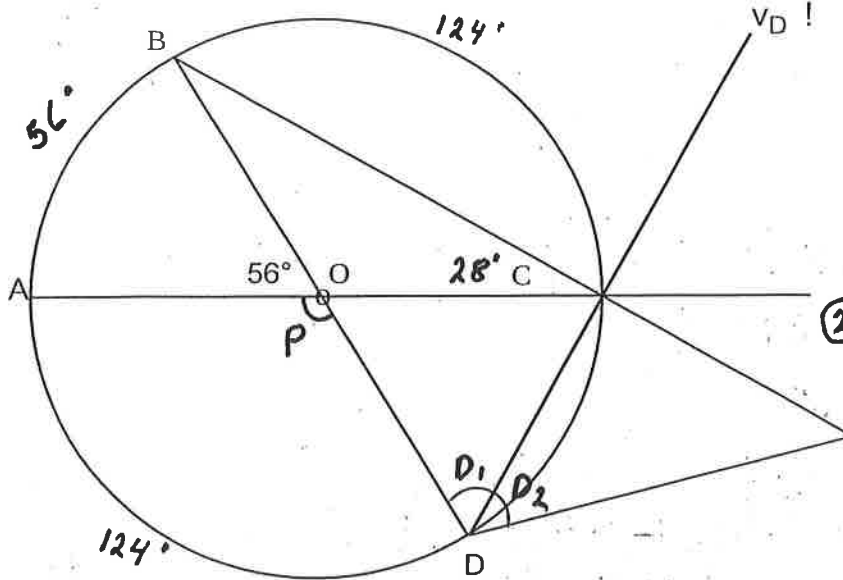
$\frac{\widehat{AE} - \widehat{BC}}{2} = D$

$\frac{\widehat{AE} - 63^\circ}{2} = 49^\circ \cdot 2$

$\widehat{AE} - 63^\circ = 98^\circ$
 $+63^\circ \quad +63^\circ$

$\widehat{AE} = 161^\circ$

3. (20%) Finndu stærð þeirra horna og boga sem beðið er um. Punkturinn O er miðja hringsins.



Boginn $\widehat{AB} = 56^\circ$

Boginn $\widehat{AD} = 124^\circ$

Hornið D = 124°

Hornið C = 28°

④ Finnum D:

boginn $\widehat{BC} = \text{boginn } \widehat{AD} = 124^\circ$

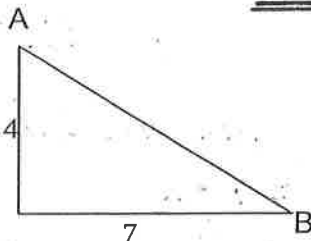
þar sem $O = P = 124^\circ$ Tophorn

D_1 er ferilhörn = $BC/2 = 124/2 = 62^\circ$

4. (5%) Finndu stærð hornanna A og B á rétthyrnda þríhyrningnum sem vantar á myndinni hér að neðan. Sýndu formúlu.

VD skiptir horni D í 2 jafn stór horn

$D_1 = D_2 = 62^\circ \rightarrow D = 2 \cdot 62^\circ = 124^\circ$



① $\tan(A) = \frac{\text{möttæg hlid}}{\text{adlæg hlid}}$

$\tan(A) = \frac{7}{4} \rightarrow$

$A = \tan^{-1}(7/4) = 60,255^\circ$

$A \approx 60,26^\circ$

② $\tan(B) = \frac{\text{möttæg hlid}}{\text{adlæg hlid}}$

$\tan(B) = \frac{4}{7} \rightarrow$

$B = \tan^{-1}(4/7) = 29,744 \approx \underline{29,74^\circ}$

① Finnum \widehat{AB}

Bogalengdin \widehat{AB} er jafnt og miðhornið 56° sem spannar sömu bogalengd \widehat{AB}
 $\widehat{AB} = \text{miðhorn} = 56^\circ$

② Finnum C sem er ferilhörn:

$C = \frac{\widehat{AB}}{2} = \frac{56^\circ}{2} = \underline{28^\circ}$

Ferilhornið spannar sömu bogalengd og miðhornið en er helmingi minna

③ Finnum \widehat{AD} :

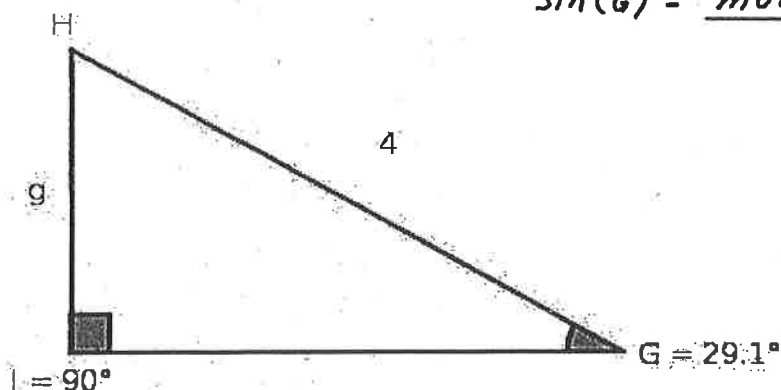
$P + 56^\circ = 180^\circ$ grannhorn á beinni líni

$P = 180^\circ - 56^\circ = 124^\circ$

$O = P = 124^\circ$ Tophorn eru jafnstór

$\widehat{AD} = P = 124^\circ$ Bogalengdin \widehat{AD} er jafnt og miðhornið $P = 124^\circ$ sem spannar sömu bogalengd \widehat{AD}

5. (5%) Reiknaðu út lengd hliðar g. Hliðin i er 4 og hornið G er $29,1^\circ$. Sýndu formúlu.



$$\sin(G) = \frac{\text{mótlæg skammhlid}}{\text{langhlid}}$$

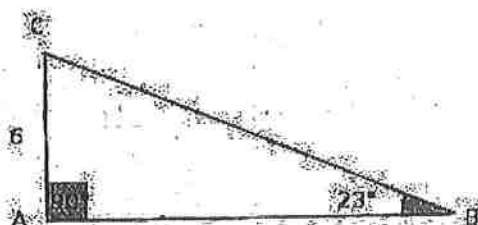
$$4 \cdot \sin(29,1^\circ) = \frac{g}{4} \cdot 4$$

$$g = 4 \cdot \sin(29,1^\circ) = 1,945 \approx \underline{\underline{1,95}}$$

6. (10%) Reiknaðu út lengdina á hlið a og hlið c. Hornið B er 23° og hliðin b er 6.

Hlið a = 15,36

Hlið c = 14,14



- ① Finnum hliðina a:

$$\sin(B) = \frac{\text{mótlæg skammhlid}}{\text{langhlid}}$$

$$a \cdot \sin(23^\circ) = \frac{6}{a} \cdot a$$

$$a \cdot \sin(23^\circ) = \frac{6}{\sin(23^\circ)}$$

$$a = \frac{6}{\sin(23^\circ)} = 15,355 \approx \underline{\underline{15,36}}$$

- ② Finnum hliðina c:

$$\cos(23^\circ) = \frac{\text{adlag skammhlid}}{\text{langhlid}}$$

$$\cos(23^\circ) = \frac{c}{15,36} \rightarrow$$

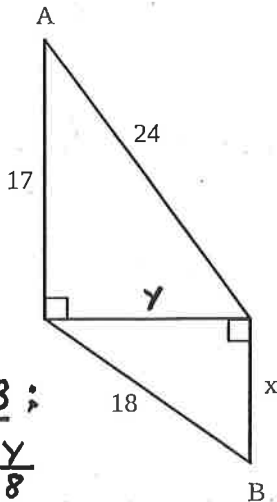
$$c = 15,36 \cdot \cos(23^\circ) = 14,138 \approx \underline{\underline{14,14}}$$

eda $\tan(B) = \frac{\text{mótlæg hlid}}{\text{adlag hlid}}$

$$c \cdot \tan(23^\circ) = \frac{6}{c} \cdot c$$

$$c = \frac{6}{\tan(23^\circ)} = 14,135 \approx \underline{\underline{14,16}}$$

7. (15%)



① Finnum A:

$$\cos(A) = \frac{17}{24}$$

$$A = \cos^{-1}(17/24)$$

$$A \approx \underline{44,9^\circ}$$

Reiknaðu:

a) Hornið A: 44,9°

b) Hliðina x: 6,09

c) Hornið B: _____

② Finnum y:

$$a^2 + b^2 = c^2 \text{ Pýþagóras}$$

$$y^2 + 17^2 = 24^2$$

$$y^2 + 289 = 576$$

$$y^2 = 576 - 289 = 287$$

$$y = \sqrt{287} \approx \underline{16,94}$$

④ Finnum B:

$$\sin(B) = \frac{y}{18}$$

$$\sin(B) = \frac{16,94}{18}$$

$$B = \sin^{-1}(16,94/18)$$

$$B \approx \underline{70,24^\circ}$$

③ Finnum x: $a^2 + b^2 = c^2$ Pýþagóras

$$x^2 + y^2 = 18^2$$

$$x^2 + 16,94^2 = 18^2$$

$$x^2 = 18^2 - 16,94^2 = 37,0364$$

$$x = \sqrt{37,0364}$$

$$x \approx \underline{6,09}$$

8. (20%) Þríhyrningurinn ABC er ekki rétthyrndur.

Hornið B = 47°, hliðin a = 6 og hliðin c = 11.

Reiknaðu:

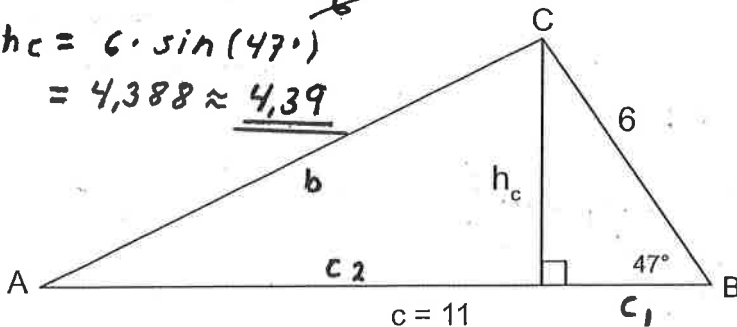
$h_c = \underline{4,39}$

① Finnum h_c :

$$\sin(B) = \frac{\text{mótlæg hlið}}{\text{langhlið}}$$

$$6 \cdot \sin(47^\circ) = \frac{h_c}{11}$$

$$h_c = 6 \cdot \sin(47^\circ) = 4,388 \approx \underline{4,39}$$



hornið A = 32,43°

hliðina b = 8,19

hliðina n_c = _____

⑤ Finnum b:

$$a^2 + b^2 = c^2 \text{ Pýþagóras}$$

$$h_c^2 + c_2^2 = b^2$$

$$4,39^2 + 6,91^2 = b^2$$

$$b^2 = 67,0202$$

$$b = \sqrt{67,0202} \approx \underline{8,19}$$

② Finnum c_1 :

$$6 \cdot \cos(47^\circ) = \frac{c_1}{6}$$

$$c_1 = 6 \cdot \cos(47^\circ) = 4,091 \approx \underline{4,09}$$

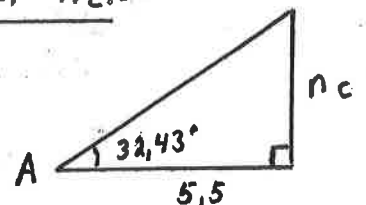
③ $c_2 = c - c_1 = 11 - 4,09 = \underline{6,91}$

④ Finnum A:

$$\tan(A) = \frac{h_c}{c_2} = \frac{4,39}{6,91}$$

$$A = \tan^{-1}(4,39/6,91) = 32,428^\circ \approx \underline{32,43^\circ}$$

⑥ Finnum n_c :



n_c skiptir hliðinni c sem er 11 í 2 jafnstórar lengdir $11/2 = 5,5$

$$\tan(32,43^\circ) = \frac{n_c}{5,5}$$

$$n_c = 5,5 \cdot \tan(32,43^\circ) \approx \underline{3,49}$$