



Kaflapróf 1

STÆF2RH05

Kafla 1 - 3

Haustönn 2018

Nafn:

Lausn

Einkunn:

1. (15%) Línan n er samsíða AB . Finndu stærð hornanna:

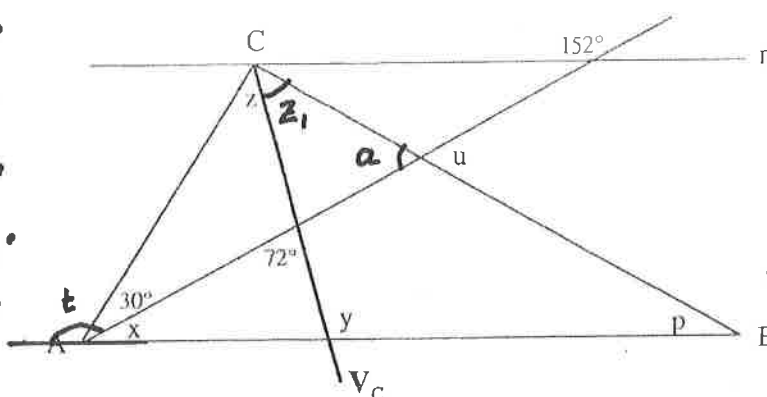
$p = 38^\circ$

$u = 66^\circ$

$x = 28^\circ$

$y = 100^\circ$

$z = 42^\circ$



① $t + 30^\circ$ og 152°
eru einslæg horn
 $t + 30^\circ = 152^\circ$
 $-30^\circ \quad -30^\circ$

 $t = 122^\circ$

② t og 30° og x
eru grannhorn
 $t + 30^\circ + x = 180^\circ$
 $122 + 30^\circ + x = 180^\circ$
 $152^\circ + x = 180^\circ$
 $-152 \quad -152$

 $x = 28^\circ$

2. (10%) Hér til hliðar er reglulegur marghyrningur.

- a) Hvað eru horn hans mörg?
- b) Hver er hornasumma hans?
- c) Hvað er hvert horn hans stórt?

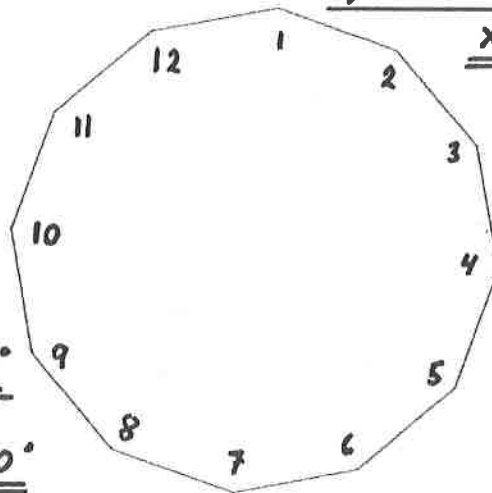
Fjöldi horna: 12

$(n-2) \cdot 180^\circ = (12-2) \cdot 180^\circ$

Hornasumma: $10 \cdot 180^\circ = 1800^\circ$

Stærð horns = $\frac{\text{hornasumma}}{n}$

Stærð hvers horns: $\frac{1800}{12} = 150^\circ$



1. ③

$z + 30^\circ = 72^\circ$ utanvert horn er jafnt og summa mótlegtra innhorna
 $-30^\circ \quad -30^\circ$

$z = 42^\circ$

④

$y = x + 72^\circ$ utanvert horn

$y = 28^\circ + 72^\circ = 100^\circ$

⑤

$z_1 = z = 42^\circ$ helmingahorn eru jafnstór

⑥

$p + x + 30^\circ + z + z_1 = 180^\circ$ hornasumma

$p + 28^\circ + 30^\circ + 42^\circ + 42^\circ = 180^\circ$ þríhyrnings

$p + 142^\circ = 180^\circ$

$-142 \quad -142$

$p = 38^\circ$

⑦

hornasumma þríh.

$a + 30^\circ + z + z_1 = 180^\circ$

$a + 30^\circ + 42^\circ + 42^\circ = 180^\circ$

$a + 114^\circ = 180^\circ$

$-114 \quad -114$

$a = 66^\circ$

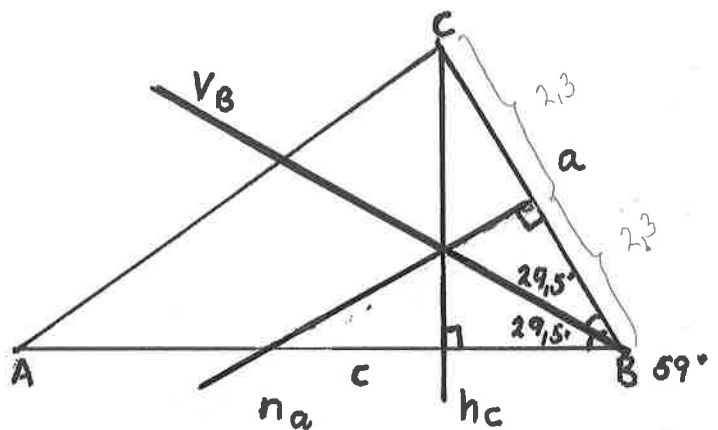
⑧

Topp horn eru jafnstór

$u = a = 66^\circ$

3. (15%) Hér er þríhyrningurinn ABC. Teiknaðu eftirfarandi línur inn á þríhyrninginn.

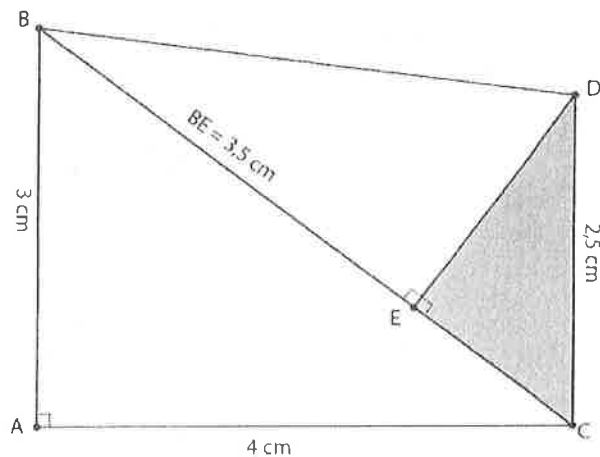
- Hæðina á c (h_c).
- Miðþveril á a (n_a).
- Helmingunnarlínu hornsins B (V_B).



4. (20%) Hér til hliðar eru þrjú rétthyrndir þríhyrningar. Hliðin $AB = 3$ cm, $AC = 4$ cm, $CD = 2,5$ cm og $BE = 3,5$ cm. Notaðu reglu Pýþagóras til að finna

a) Hliðina CE. 1,5 cm

b) Hliðina DE. 2 cm



① Jafna Pýþagórasar

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 AB^2 + AC^2 &= BC^2 \\
 3^2 + 4^2 &= BC^2 \\
 9 + 16 &= BC^2 \\
 25 &= BC^2 \\
 BC &= \sqrt{25} = 5 \text{ cm}
 \end{aligned}$$

② $BE + CE = BC = 5$

$$\begin{aligned}
 3,5 + CE &= 5 \\
 \underline{-3,5} \quad \underline{-3,5} & \\
 CE &= \underline{\underline{1,5 \text{ cm}}}
 \end{aligned}$$

③ Jafna Pýþagórasar

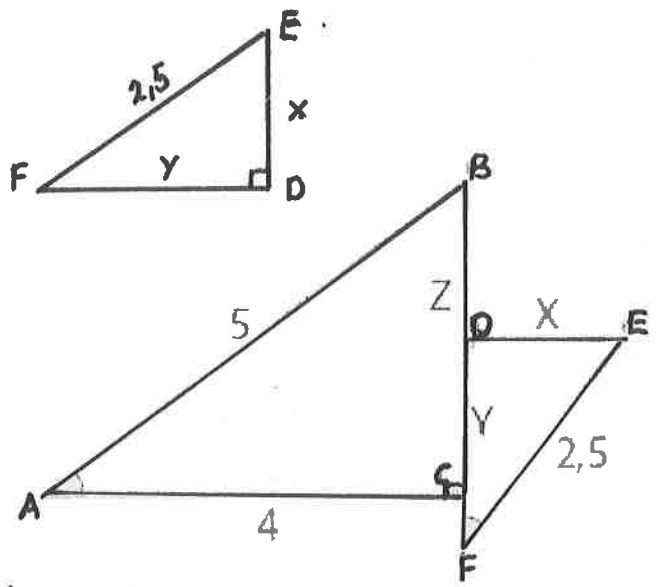
$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 DE^2 + CE^2 &= CD^2 \\
 DE^2 + (1,5)^2 &= (2,5)^2 \\
 DE^2 + 2,25 &= 6,25 \\
 \underline{-2,25} \quad \underline{-2,25} &
 \end{aligned}$$

$$DE^2 = 4$$

$$DE = \sqrt{4}$$

$$\underline{\underline{DE = 2 \text{ cm}}}$$

5. (10%) Þríhyrningarnir ABC og DEF eru einshyrndir. Hornin A og F eru jafnstór. $X = DE$, $Y = DF$. Finndu



a) X: 1,5

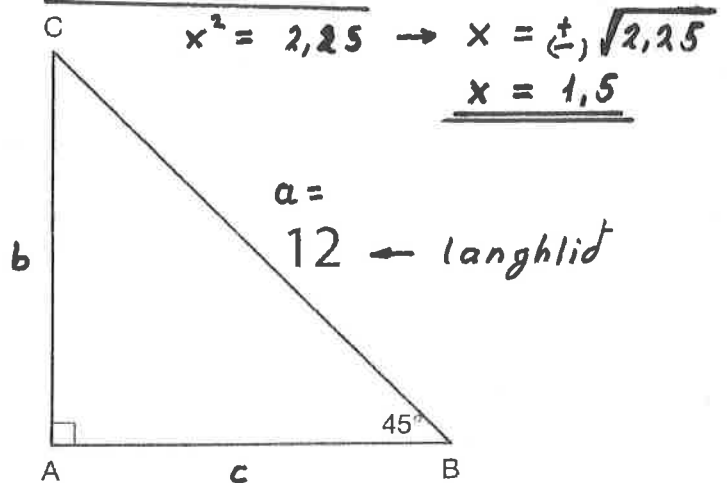
b) Y: 2

① $\frac{Y}{4} = \frac{2,5}{5}$
 $Y = \frac{4 \cdot 2,5}{5} = \frac{10}{5}$
 $Y = 2$

② Finna X
 Pýþagóras
 $x^2 + y^2 = (2,5)^2$
 $x^2 + 2^2 = 6,25$
 $x^2 + 4 = 6,25$
 $\quad -4 \quad -4$

6. (5%) Þríhyrningur er $(45^\circ, 45^\circ, 90^\circ)$. Langhlið hans er 12.

Sýndu með útreikningum að lengd skammhliðar $b = 6\sqrt{2}$.



Þríhyrningurinn er jafnarma → skammhliðarnar eru jafnstórar

$b = c = x$

Jafna Pýþagórasar

$b^2 + c^2 = a^2$

$x^2 + x^2 = 12^2$

$\frac{2x^2}{2} = \frac{144}{2}$

$x^2 = 72$

$x = \sqrt{72} = \sqrt{8 \cdot 9}$

$x = \sqrt{8} \cdot \sqrt{9} = \sqrt{4 \cdot 2} \cdot 3$

$x = \sqrt{4} \cdot \sqrt{2} \cdot 3 = 2 \cdot \sqrt{2} \cdot 3$

$x = 2 \cdot 3 \cdot \sqrt{2} = \underline{6\sqrt{2}}$

$b = x = 6\sqrt{2}$

Önnur aðferð:

Langhlið = mótlæg skammhlið $\cdot \sqrt{2}$

$(c = a\sqrt{2})$

$a = b\sqrt{2}$

$12 = b\sqrt{2} \leftrightarrow$

$\frac{b\sqrt{2}}{\sqrt{2}} = \frac{12}{\sqrt{2}}$ deili í jöfnuna með $\sqrt{2}$

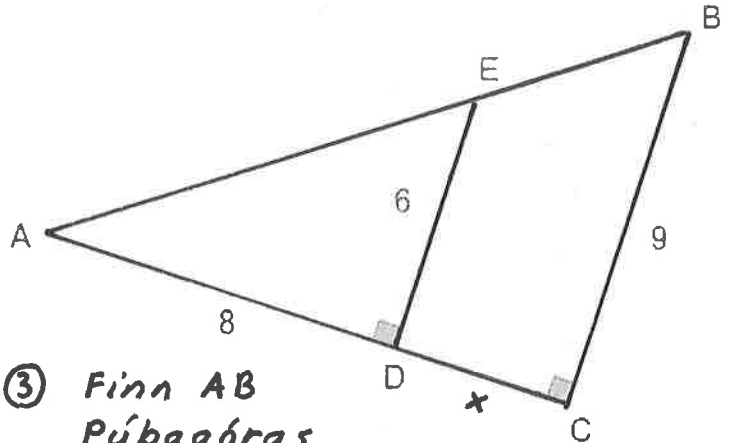
$b = \frac{12}{\sqrt{2}}$ lengi brotið með $\sqrt{2}$

$b = \frac{12 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{12\sqrt{2}}{2}$

$b = 6\sqrt{2}$

7. (10%) Þríhyrningarnir eru einslaga. $BC = 9$, $DE = 6$ og $AD = 8$.
Reiknaðu lengdina á AB og DC .

$$\begin{aligned} \textcircled{1} \quad \frac{(8+x)}{8} &= \frac{9}{6} \\ 8+x &= \frac{72}{6} \\ 8+x &= 12 \\ -8 & \quad -8 \\ \hline x &= DC = 4 \end{aligned}$$

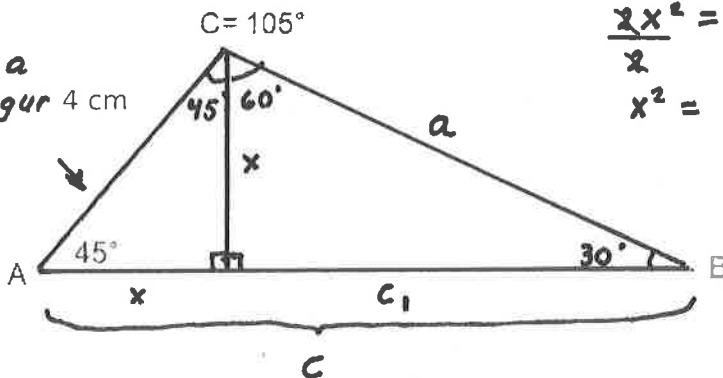


$$\begin{aligned} \textcircled{2} \quad AC &= 8 + DC = 8 + 4 = 12 \\ AB &= 15 \\ DC &= 4 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad &\text{Finn } AB \\ &\text{Pýþagóras} \\ &AB^2 = BC^2 + AC^2 \\ &AB^2 = 9^2 + 12^2 = 81 + 144 \\ &AB^2 = 225 \\ &AB = \sqrt{225} = 15 \end{aligned}$$

8. (15%) Í þríhyrningnum ABC er $A = 45^\circ$, $C = 105^\circ$ og $b = 4$ cm.
Finndu með stærðfræðilegri nákvæmni hliðina a , hliðina c og
flatarmál þríhyrningsins.

Jafnarma
þríhyrningur



$$a = 4\sqrt{2} \text{ cm}$$

$$c = (2\sqrt{2} + 2\sqrt{6}) \text{ cm}$$

$$\text{flatarmál} = (4 + 4\sqrt{3}) \text{ cm}^2$$

$$\begin{aligned} \textcircled{4} \quad c &= x + c_1 = 2\sqrt{2} + 2\sqrt{6} \\ c &= (2\sqrt{2} + 2\sqrt{6}) \text{ cm} \end{aligned}$$

$$\textcircled{5} \quad F = \frac{g \cdot h}{2} = \frac{c \cdot x}{2} = \frac{(2\sqrt{2} + 2\sqrt{6}) \cdot 2\sqrt{2}}{2} = (2\sqrt{2} + 2\sqrt{6}) \cdot \sqrt{2}$$

$$F = 2\sqrt{2} \cdot \sqrt{2} + 2\sqrt{6} \cdot \sqrt{2} = 2 \cdot 2 + 2\sqrt{12} = 4 + 2\sqrt{4 \cdot 3}$$

$$F = 4 + 2\sqrt{4 \cdot 3} = 4 + 2 \cdot 2\sqrt{3} = 4 + 4\sqrt{3} = (4 + 4\sqrt{3}) \text{ cm}^2$$

$$\begin{aligned} \textcircled{1} \quad &\text{Pýþagóras} \\ &x^2 + x^2 = 4^2 \\ &\frac{2x^2}{2} = \frac{16}{2} \\ &x^2 = 8 \end{aligned}$$

$$\begin{aligned} x &= \sqrt{8} \\ x &= \sqrt{4 \cdot 2} \\ x &= 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad &\text{þríhyrningur með} \\ &\text{hornin } 30^\circ - 90^\circ - 60^\circ \\ &\text{sérstakur 2} \\ &\text{Langhlið} = 2 \cdot \text{styttri} \\ &\text{skammhlið} \\ &a = 2 \cdot x = 2 \cdot 2\sqrt{2} \\ &a = 4\sqrt{2} \text{ cm} \end{aligned}$$

$$\textcircled{3} \quad \text{Lengri skammhlið} = \text{styttri skammhlið} \cdot \sqrt{3}$$

$$c_1 = x\sqrt{3} = 2\sqrt{2} \cdot \sqrt{3}$$

$$c_1 = 2\sqrt{2 \cdot 3} = 2\sqrt{6}$$