

Hornafræði

Skilgreiningar

Sínus: $\sin(v)$

Cosínus: $\cos(v)$

Tangens: $\tan(v)$

Cotangens: $\cot(v)$

$$\cos(v) = \frac{\text{aðlæg hlið}}{\text{langhlið}}$$

$$\sin(v) = \frac{\text{mótlæg hlið}}{\text{langhlið}}$$

$$\tan(v) = \frac{\text{mótlæg hlið}}{\text{aðlæg hlið}} = \frac{1}{\cot(v)} = \frac{\sin(x)}{\cos(x)}$$

Umskriftarreglur

$$\sin(-v) = -\sin(v)$$

$$\cos(-v) = \cos(v)$$

$$\tan(-v) = -\tan(v)$$

$$\cot(-v) = -\cot(v)$$

$$\sin(v+\pi) = -\sin(v)$$

$$\cos(v+\pi) = -\cos(v)$$

$$\tan(v+\pi) = \tan(v)$$

$$\cot(v+\pi) = \cot(v)$$

$$\sin(\pi-v) = \sin(v)$$

$$\cos(\pi-v) = -\cos(v)$$

$$\tan(\pi-v) = -\tan(v)$$

$$\cot(\pi-v) = -\cot(v)$$

$$\sin(v+\pi/2) = \cos(v)$$

$$\cos(v+\pi/2) = -\sin(v)$$

$$\tan(v+\pi/2) = -\cot(v)$$

$$\cot(v+\pi/2) = -\tan(v)$$

$$\sin(v-\pi/2) = -\cos(v)$$

$$\cos(v-\pi/2) = \sin(v)$$

$$\tan(v-\pi/2) = -\cot(v)$$

$$\cot(v-\pi/2) = -\tan(v)$$

$$\sin(\pi/2-v) = \cos(v)$$

$$\cos(\pi/2-v) = \sin(v)$$

$$\tan(\pi/2-v) = \cot(v)$$

$$\cot(\pi/2-v) = \tan(v)$$

$$\cos(v) = \frac{\pm 1}{\sqrt{1 + \tan^2(v)}}$$

$$\sin(v) = \frac{\pm \tan(v)}{\sqrt{1 + \tan^2(v)}}$$

$$\sin^2(v) + \cos^2(v) = 1$$

$$\frac{1}{\cos^2(v)} = 1 + \tan^2(v)$$

$$\frac{1}{\sin^2(v)} = 1 + \cot^2(v)$$

Tvöfalt horn

$$\sin(2v) = 2 \sin(v) \cdot \cos(v)$$

$$\cos(2v) = 2 \cos^2(v) - 1$$

$$\cos(2v) = 1 - 2 \sin^2(v)$$

$$\cos(2v) = \cos^2(v) - \sin^2(v)$$

$$\tan(2v) = \frac{2 \tan(v)}{1 - \tan^2(v)}$$

$$\cos^2(v) = \frac{1 + \cos(2v)}{2}$$

$$\sin^2(v) = \frac{1 - \cos(2v)}{2}$$

Hálft horn

$$\cos\left(\frac{v}{2}\right) = \pm \sqrt{\frac{1 + \cos(v)}{2}}$$

$$\sin\left(\frac{v}{2}\right) = \pm \sqrt{\frac{1 - \cos(v)}{2}}$$

$$\tan\left(\frac{v}{2}\right) = \pm \sqrt{\frac{1 - \cos(v)}{1 + \cos(v)}}$$

Summuformúlur

$$\sin(u+v) = \sin(u) \cos(v) + \cos(u) \sin(v),$$

$$\sin(u-v) = \sin(u) \cos(v) - \cos(u) \sin(v)$$

$$\cos(u+v) = \cos(u) \cos(v) - \sin(u) \sin(v),$$

$$\cos(u-v) = \cos(u) \cos(v) + \sin(u) \sin(v),$$

$$\tan(u+v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u) \tan(v)},$$

$$\tan(u-v) = \frac{\tan(u) - \tan(v)}{1 + \tan(u) \tan(v)}$$

Kósínusreglan

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$

Sínusreglan

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} = 2R$$

Flatarmál þríhyrnings

$$F_{ABC} = \frac{1}{2} b \cdot c \cdot \sin(A)$$

Gráður í radíana, radíanar í gráður

$$v^\circ = \frac{v \cdot \pi}{180} \text{ rad} \quad v \text{ rad} = \frac{v \cdot 180^\circ}{\pi}$$

Lausn á hornafallajöfnu

$$\sin(v) = k$$

$$\cos(v) = k$$

$$\tan(v) = k$$

$$v = \begin{cases} \sin^{-1}(k) + 360 \cdot h \\ 180 - \sin^{-1}(k) + 360 \cdot h \end{cases}$$

$$v = \begin{cases} \cos^{-1}(k) + 360 \cdot h \\ -\cos^{-1}(k) + 360 \cdot h \end{cases}$$

$$v = \tan^{-1}(k) + 180 \cdot h$$

Vigrar

Ef $A=(x_1, y_1)$ og $B=(x_2, y_2)$ þá er $\overline{AB}=\begin{pmatrix} x_2-x_1 \\ y_2-y_1 \end{pmatrix}$

Ef $\vec{a}=\begin{pmatrix} x \\ y \end{pmatrix}$ þá er **hallatala** $h_a=\frac{y}{x}$, **lengd** $|\vec{a}|=\sqrt{x^2+y^2}$ **þvervigur** $\vec{a}_\perp=\begin{pmatrix} -y \\ x \end{pmatrix}$ og $t\vec{a}=\begin{pmatrix} t\cdot x \\ t\cdot y \end{pmatrix}$

Einingarvigrar

$$\vec{e}=\frac{1}{|\vec{a}|}\cdot\vec{a}=\begin{pmatrix} \frac{x}{|\vec{a}|} \\ \frac{y}{|\vec{a}|} \end{pmatrix} \quad \vec{e}\cdot|\vec{a}|=\vec{a} \quad \vec{e}_v=\begin{bmatrix} \cos(v) \\ \sin(v) \end{bmatrix} \quad \vec{i}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \vec{j}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Stefnuhorn vigrs

1. fjórðungur

2. fjórðungur

3. fjórðungur

4. fjórðungur

$$v=\tan^{-1}\left(\frac{y}{x}\right)$$

$$v=\tan^{-1}\left(\frac{y}{x}\right)+180^\circ$$

$$v=\tan^{-1}\left(\frac{y}{x}\right)+180^\circ$$

$$v=\tan^{-1}\left(\frac{y}{x}\right)+360^\circ$$

Ef $\vec{a}=\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ og $\vec{b}=\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ þá er **summan** $\vec{a}+\vec{b}=\begin{pmatrix} x_1+x_2 \\ y_1+y_2 \end{pmatrix}$ **Innfeldi** $\vec{a}\cdot\vec{b}=x_1\cdot x_2+y_1\cdot y_2$
og $\vec{a}\perp\vec{b}\Leftrightarrow\vec{a}\cdot\vec{b}=0$

$$|\vec{a}|^2=\vec{a}\cdot\vec{a} \quad |\vec{a}+\vec{b}|^2=|\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2$$

$$|\vec{a}-\vec{b}|^2=|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$$

Lausn jöfnuhneppis

$$a_1x+b_1y=c_1$$

$$a_2x+b_2y=c_2$$

$$x=\frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad y=\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

D - regla

$$Ax^2+Bx+C=0$$

$$D=B^2-4AC, \quad x=\frac{-B\pm\sqrt{D}}{2A}$$

Horn milli vigra: $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos(v)$, $\vec{a}_\perp\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\sin(v)$

Almenn jafna hrings

$$(x-h)^2+(y-k)^2=r^2$$

Sporbaugur

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$F=(h\pm c, k)$$

$$c=\sqrt{a^2-b^2}$$

$$e=\frac{c}{a}$$

Breiðbogi

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$$V=(h\pm a, k)$$

Almenn jafna línu

$$ax+by+c=0$$

$$c=-ax_0-by_0$$

$$\vec{n}\cdot\overline{P_0P}=\begin{pmatrix} a \\ b \end{pmatrix}\cdot\begin{pmatrix} x-x_0 \\ y-y_0 \end{pmatrix}=0$$

Normal,

þvervigur

$$\vec{n}=\begin{pmatrix} a \\ b \end{pmatrix}$$

Fjarlægð punkts frá línu

$$d_{(l,P)}=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$

Miðpunktur línu:

Ef $A=(x_1, y_1)$ og $B=(x_2, y_2)$

$$\text{er } M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Gildistafla hornafalla

v (rad)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
v° (gráður)	0	30	45	60	90
sin(v)	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos(v)	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan(v)	0	$\frac{\sqrt{3}}{3}=\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	óskilgr.